1. Obliczyć całki nieoznaczone

(a) $\int \frac{x^2-\sqrt{x}}{\sqrt[3]{x}}dx$.

(b) $\int \sqrt[4]{3^x}dx$.

(c) $\int \operatorname{ctg}^2 x dx$.

(d) $\int \frac{e3x-1}{e^x-1}dx$.

(e) $\int \sin^2 \frac{x}{2}dx$.

(f) $\int \sqrt{x\sqrt{x\sqrt{x}}}dx$.

2. Obliczyć całki (przez części).

(a) $\int \operatorname{arc\,tg} x \ dx$.

(b) $\int x^2 \operatorname{arc\,tg} x dx$.

(c) $\int \arcsin x dx$.

(d) $\int x \sin x \cos x dx$.

(e) $\int x \ln x^2 dx$.

(f) $\int \frac{\ln x}{x^2}dx$.

(g) $\int \frac{x}{\sin^2 x}dx$.

(h) $\int e^x \cos x dx$.

3. Obliczyć całki (przez podstawienie).

(a) $\int \frac{e^{3x}}{1+e^{6x}}dx$.

(b) $\int x\sqrt{x-3}dx$.

(c) $\int \frac{x^3}{\sqrt{(1-x^2)^3}}dx$.

(d) $\int \frac{\cos \ln x}{x}dx$.

(e) $\int x\sqrt{x^2+1}dx$.

(f) $\int \frac{\sin x}{3+2\cos x}dx$.

(g) $\int \frac{e^{-4x}dx}{\sqrt{4+e^{-4x}}}$.

(h) $\int \frac{dx}{x\sqrt{x^2-2}}$.

(i) $\int \frac{x^3 dx}{\sqrt{1-x^8}}$.

(j) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$.

(k) $\int e^{\sqrt{x}} dx$.

(l) $\int \frac{\sqrt{x} dx}{x-1}$.

4. Obliczyć całki (rozkład na ułamki proste).

(a) $\int \frac{(x^2-5x+9)dx}{x^2+5x+6}$.

(b) $\int \frac{x}{(x^2+2)^2}$.

(c) $\int \frac{(x^3+x+1)dx}{x^4+x^2}$.

(d) $\int \frac{x^2}{x+1} dx$.

(e) $\int \frac{dx}{x^4+4}$.

(f) $\int \frac{dx}{x(x+1)^2}$.

(g) $\int \frac{x^2}{(x-1)^3}$.

Odpowiedzi.

1. (a) $\frac{3}{8}\sqrt[3]{x^8} - \frac{6}{7}\sqrt[6]{x^7} + C$.

(b) $\frac{4\sqrt[4]{3}^x}{\ln 3} + C$.

(c) $\operatorname{ctg} x - x + C$.

(d) $\frac{1}{2}e^{2x} + e^x + x + C$.

(e) $\frac{1}{2}x - \frac{1}{2}\sin x + C$.

(f) $\frac{8}{15}\sqrt[8]{x^{15}} + C$.

2.

(a) $x \operatorname{arc\,tg} x - \frac{1}{2}\ln(1+x^2) + C$.

(b) $\frac{1}{3}x^3 \operatorname{arc\,tg} x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1+x^2) + C$.

(c) $x \arcsin x + \sqrt{1-x^2} + C$.

(d) $-\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$.

(e) $\frac{1}{2}x^2\ln^2 x - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C$.

(f) $-\frac{1}{x}(\ln x + 1) + C$.

(g) $-x \operatorname{ctg} x + \ln|\sin x| + C$.

(h) $\frac{1}{2}e^x(\cos x + \sin x) + C$.

3.

(a) $\frac{1}{3}\operatorname{arc\,tg} e^{3x} + C$.

(b) $\frac{2}{5}(x-3)^2\sqrt{x-3} + 2(x-3)\sqrt{x-3} + C$.

(c) $\frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} + C$.

(d) $\sin\ln x + C$.

(e) $\frac{1}{3}(x^2+1)\sqrt{x^2+1} + C$.

(f) $-\frac{1}{2}\ln(3+2\cos x) + C$.

(g) $-\frac{\sqrt{4+e^{-4x}}}{2} + C$.

(h) $\frac{1}{\sqrt{2}}\operatorname{arc\,tg}\sqrt{\frac{x^2-2}{2}} + C$.

(i) $\frac{1}{4}\arcsin x^4 + C$.

(j) $2(\sqrt[6]{x}+1)^3 - 9(\sqrt[6]{x}+1)^2 + 18(\sqrt[6]{x}+1) - 6\ln(\sqrt[6]{x}+1) + C$.

(k) $2e^{\sqrt{x}}(\sqrt{x}-1) + C$.

(l) $2\sqrt{x} + \ln\frac{\sqrt{x}-1}{\sqrt{x}+1} + C$.

4. (a) $x + 23\ln|x+2| - 33\ln|x+3| + C$.

(b) $-\frac{1}{2(x^2+2)} + C$.

(c) $\ln|x| - \frac{1}{x} - \operatorname{arc\,tg} x + C$.

(d) $-\frac{1}{3}\ln|x+1| + \frac{1}{6}\ln(x^2-x+1) + 2\frac{\sqrt{3}}{3}\operatorname{arc\,tg}\frac{2x-1}{\sqrt{3}} + C$.

(e) $\frac{1}{16}\left(\ln\frac{x^2+2x+2}{x^2-2x+2+2} + \operatorname{arc\,tg}(x+1) + 2\operatorname{arc\,tg}(x-1)\right) + C$.

(f) $\ln|\frac{x}{x+1}| + \frac{1}{x+1} + C$.

(g) $\ln|x-1| - \frac{2}{x-1} - \frac{1}{2(x-1)^2} + C$.